

Detection of vorticity in Bose-Einstein condensed gases by matter-wave interference

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A phase-slip in the fringes of an interference pattern is an unmistakable characteristic of vorticity. We show dramatic two-dimensional simulations of interference between expanding condensate clouds with and without vorticity. In this way, vortices may be detected even when the core itself cannot be resolved.

Bose-Einstein condensation of a dilute gas has now been accomplished for three different alkali atoms by a number of research groups [1]. Many of the properties of the gas which were calculated from mean-field theories, such as condensate density, collective excitation spectra, and the speed of sound have also been found to agree well with these theories. One of the untested predictions of mean-field theory is the quantization of circulation of the fluid, leading to quantized vortex states. In this Letter we show how such a state may be detected by using matter-wave interference. This method may allow us to follow the dynamics of states with vorticity as well.

Several ways of *creating* vortices in a Bose-Einstein condensate (BEC) have been suggested. Initial experimental attempts relying on a rotating, off-resonance laser beam were inconclusive [2]. Another idea is that vortices may appear spontaneously out of fluctuations, as a result of rapid cooling through the critical temperature [3]. Both of the foregoing methods are analogous to techniques in more traditional superfluids such as liquid ^4He . Unlike those systems, the internal states of the alkali atoms provide a convenient “handle” for the optical creation and detection of vortices. Starting with the trap ground state, vortex states can be reached via a Raman transition quickly [4,5] or adiabatically [6].

Here we focus on the *detection* of states with vorticity or non-zero winding number. Indirect detection methods have previously been suggested [7], as has detection of the spatially-dependent phase in an ionization scheme [8]. The interference of condensates released from a double-well trap [9] was the first evidence of their phase coherence. At a vortex core, the phase will always have a singularity which is independent of atomic and trap parameters. Since there is coherence over most of the condensates, we suggest that the phase slip occurring at a vortex core be used to detect it. When colliding a condensate in the ground state with a (presumed) vortex state, the vorticity is measured by counting the number of fringes which are “skipped” as they cross the core.

We briefly review the theory of the BEC far below the critical temperature beginning from the Gross-Pitaevski (GP) equation for the condensate wavefunction $\psi(\mathbf{x}, t)$,

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}\nabla^2\psi + V(\mathbf{x})\psi + \frac{u}{2}|\psi|^2\psi. \quad (1)$$

In the case of axisymmetric harmonic traps, the external potential is

$$V(r, z) = \frac{1}{2}(r^2 + \lambda^2 z^2) \quad (2)$$

Here the time is scaled in units of the reciprocal trap frequency, $\frac{1}{\omega_\perp}$, and all lengths are scaled in units of the transverse trap width,

$$a_\perp = \sqrt{\frac{\hbar}{M\omega_\perp}} \quad (3)$$

$$\lambda = \frac{\omega_\perp}{\omega_z} \quad (4)$$

so that the dimensionless interaction strength is

$$u = \frac{8\pi a N}{a_\perp} \quad (5)$$

where N is the number of condensate atoms, and a is the s -wave scattering length. Since we are interested primarily in the two-dimensional dynamics, we assume that the z dimension is thin enough that all dynamics in that direction can be neglected ($\lambda \gg 1$).

We distinguish between *pure vortex* states and generic states with *vorticity*. The latter are any states for which there exists a closed curve C such that the circulation (in unscaled variables) obeys

$$\Gamma = \oint_C \mathbf{v} \cdot d\mathbf{x} = \frac{\hbar m}{M} \neq 0. \quad (6)$$

Owing to the single-valuedness of the wavefunction, m must be an integer. The structure near the vortex core is always similar: for cylindrical coordinates (ρ, ϕ) centered at (x_c, y_c) , the density goes from 0 to n_0 within a radius of the order of the healing length

$$\xi = \frac{|m|}{\sqrt{8\pi n_0 a}}, \quad (7)$$

and the phase is

$$\phi = m\varphi = m \arctan \frac{y - y_c}{x - x_c}. \quad (8)$$

Circulation is conserved around any closed curve flowing with the condensate (Kelvin’s theorem). On the other

hand, these states are not stationary; our numerical solution of the GP equation shows precession around the trap center with a period depending on the distance from it to the core and on u . For example, when $u = 300$, a core 1.5 from the center precesses with a period ≈ 19 (Fig. 1). This appears to violate Ehrenfest's theorem; however, the precession is accompanied by considerable density excitations so that the center of mass still obeys classical equations of motion. From this we may expect that the dynamics of generic vorticity states depends not only on the atomic collisions but also damping rates of excitations.

In the case of axisymmetric traps, a subset of the *vorticity* states are *pure vortex* states of the form

$$\psi(r, z, \varphi) = f(r, z)e^{im\varphi} \quad (9)$$

These states are stationary and are eigenstates of azimuthal angular momentum $L_z = \hbar m$. The metastability of these states is still controversial [10,11]. However, one decay scenario is that pure vortex states turn into vorticity states with the core off-center [12]; the possible observation of this process is one of the motivations for our detection scheme.

For interference between a generic vorticity state and a ground state moving towards each other with relative velocity $2k$ in the x -direction, in the core region the wavefunction is approximated by

$$\psi(\mathbf{x}) \simeq \sqrt{n_1}e^{ikx} + \sqrt{n_2}e^{(-ikx+im\varphi)} \quad (10)$$

resulting in a density

$$n(\mathbf{x}) \simeq n_1 + n_2 + 2\sqrt{n_1 n_2} \cos(2kx - m\varphi) \quad (11)$$

This density displays a series of fringes, with $|m| + 1$ fringes merging into 1 at the origin, which is the location of the vortex core. Such patterns have been used to detect vortices in nonlinear optics [13]. The momentum k may be acquired from free expansion of the condensates or by moving the traps.

We have computed several interference patterns between states with and without vorticity, by numerically solving the GP equation in two dimensions. Our initial configuration consists of a pair of trapped condensate clouds with equal numbers of atoms. It is not necessary that either the actual number or the relative phase between the clouds be known. In Fig. 2, the cloud centered at $(6.25, 0)$ begins in the $m = -1$ pure vortex state while that at $(-6.25, 0)$ is the trap ground state. The traps are switched off at $t = 0$. Up until $t \simeq 1.5$ the two clouds expand independently. Then, as in the first interference experiment [9], the “pulsed” sources lead to straight fringes. However, as the core region of the upper condensate approaches the lower condensate, the difference in velocity between the upper and lower sides of the vortex tilts the fringes from the vertical. Finally, as the fringes reach the position of the core, the phase slip in fringes appears clearly (Fig. 3). We note that the core is smaller than the fringe spacing in this example.

In the second configuration, shown in Fig. 4, the cloud on the right is replaced by an $m = -1$ generic vorticity state with the trap center placed at $(-12.5, 0)$ and the core at $(x_c = 12.5, y_c = 0.75)$. This state was produced by evolving the wavefunction in imaginary time, subject to the constraints of normalization and that the phase be given by Eq. (8). The cloud on the left is the ground state centered at $(12.5, 0)$. Also, the condensates are now launched towards each other so the wavefunction is multiplied by $\exp(ikx)$ for $x < 0$ and by $\exp(-ikx)$ for $x > 0$, with $k = 4$. Since the vortex is off-center, it begins to precess before the two clouds collide. In this example, the period of this precession is much longer than the time until the interference pattern reaches the core. Thus, in Fig. 5 the position of the phase-slip above the x -axis indicates the initial position of the vortex. This suggests that precession of a core may be observed by repeatedly creating an off-center vortex, allowing it to evolve in the trap for successively longer times, and then interfering it with a ground state.

We have also examined the case of vortex states with $|m| > 1$. Any non-azimuthally symmetric perturbation is expected to break these into a collection of $m = \pm 1$ generic vortices. Fig. 6 is the pattern resulting from freely expanding clouds at $(-6.25, 0)$, $m = 0$, and $(6.25, 0)$, $m = 2$. In this case two fringes are skipped; however it is not possible to determine from the density alone that in fact we have two nearby $m = 1$ phase slips.

In an actual experiment, the core region itself may be obscured for several reasons. First, if the local density is high enough, the core radius ξ will be less than the imaging wavelength and may not be resolved. Second, light traveling along the z -direction will always pass through regions of non-zero density if the core is not exactly parallel to the laser beam. (This problem can be avoided by optically pumping atoms in thin sheets perpendicular to the imaging beam [9]). Finally, the position of the core is subject to thermal and zero-point fluctuations of the order of its radius [14]. We note that detection of the fringes avoids all of these problems. The same method may also be used to measure non-zero winding numbers in toroidal traps.

We have assumed that damping takes place on longer timescales than the dynamics seen in our simulations, which run up to a few trap periods. As mentioned above, to accurately follow the motion of generic vorticity states requires that we go beyond the GP equation to include thermal effects. We are continuing research in this direction.

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- [7] Indirect evidence for vortex states could come from frequency shifts and breaking of degeneracy in the excitation spectra [10,11]. Of course those detection methods would require knowledge of the trap parameters, number of atoms and the scattering length, which may not be accurately known. We note that there is not complete agreement on the nature of the excitation spectrum; this is also related to the question of the stability of the vortex [12].
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FIG. 1. Position of a precessing core in a generic vorticity state. *Dashed curve*, x ; *solid curve*, y coordinate. The initial state was prepared by imaginary time evolution subject to constrained normalization and phase. Interaction strength $u = 300$.

FIG. 2. Initial density of two-trap BEC for interference. Right condensate is in the $m = -1$ vortex state, left condensate is in the trap ground state. Interaction strength $u = 300$.

FIG. 3. Density at $t = 10$ after the traps of Fig. 2 have been switched off. The original vortex is clearly evident from the missing fringe near $(8, 0)$.

FIG. 4. Initial density for two condensates moving towards each other. Left cloud is the ground state, right cloud has an $m = -1$ vortex core at $(-12.5, 0.75)$. Interaction strength is $u = 300$.

FIG. 5. Density at $t = 10$ after the traps of Fig. 4 moving at $k = \pm 4$ have been switched off. The position of the phase slip is still above the x -axis, and has precessed less than $1/8$ of the way around the cloud.

FIG. 6. Density at $t = 10$ after free expansion of two trapped condensates, with right cloud starting in the $m = 2$ vortex state, left cloud in the trap ground state. Interaction strength $u = 300$.

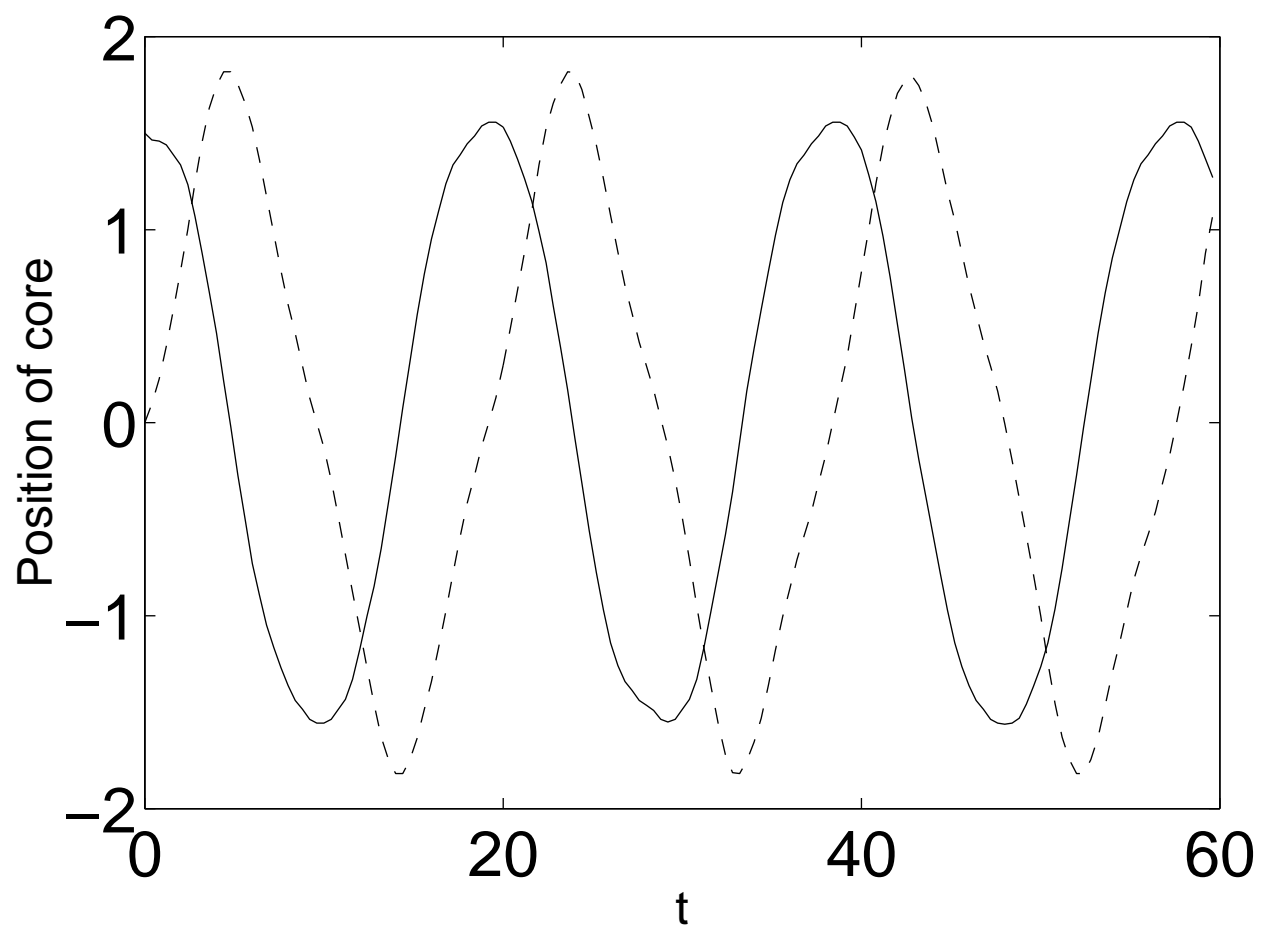


Fig. 1

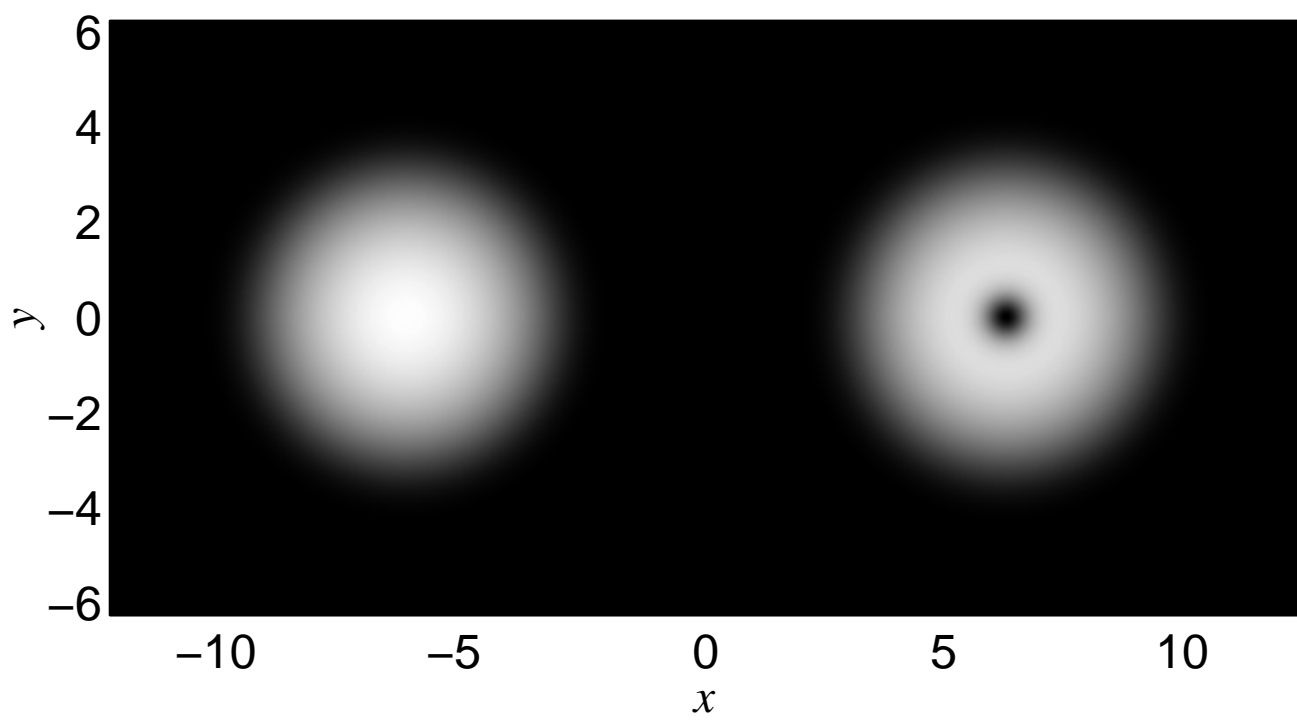


Fig. 2

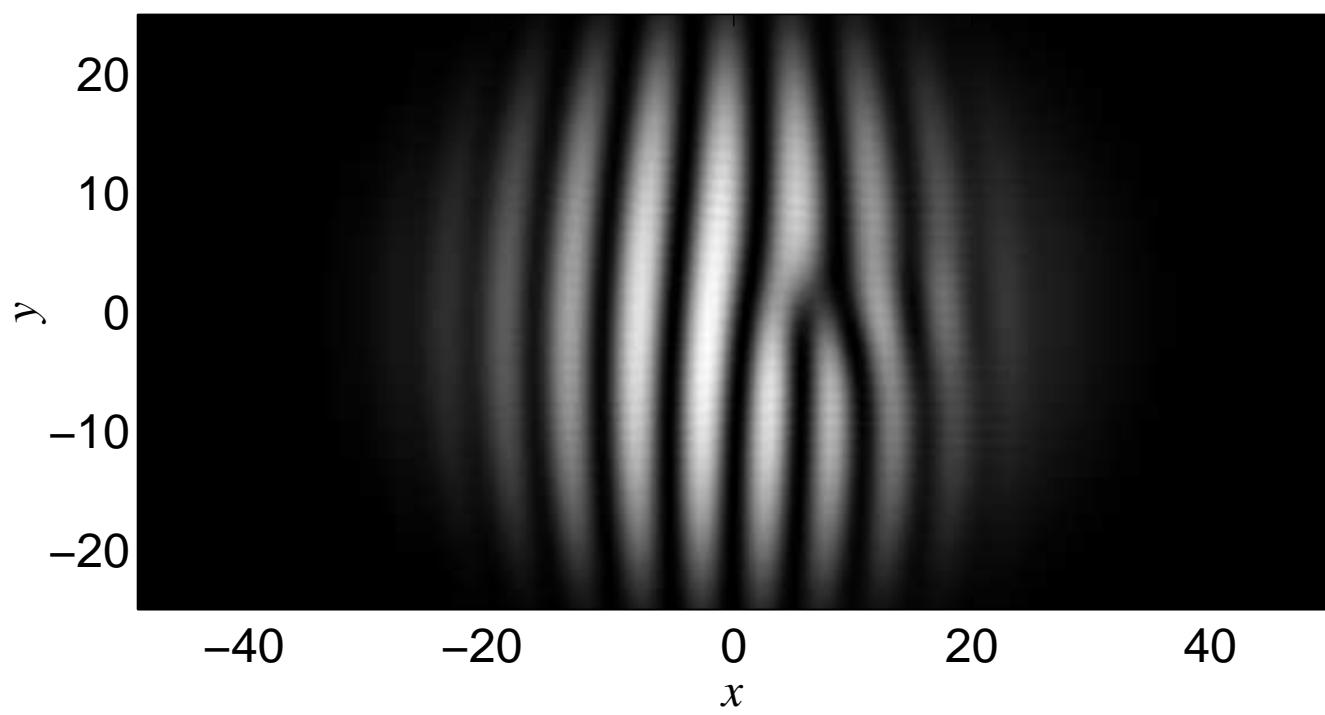


Fig. 3

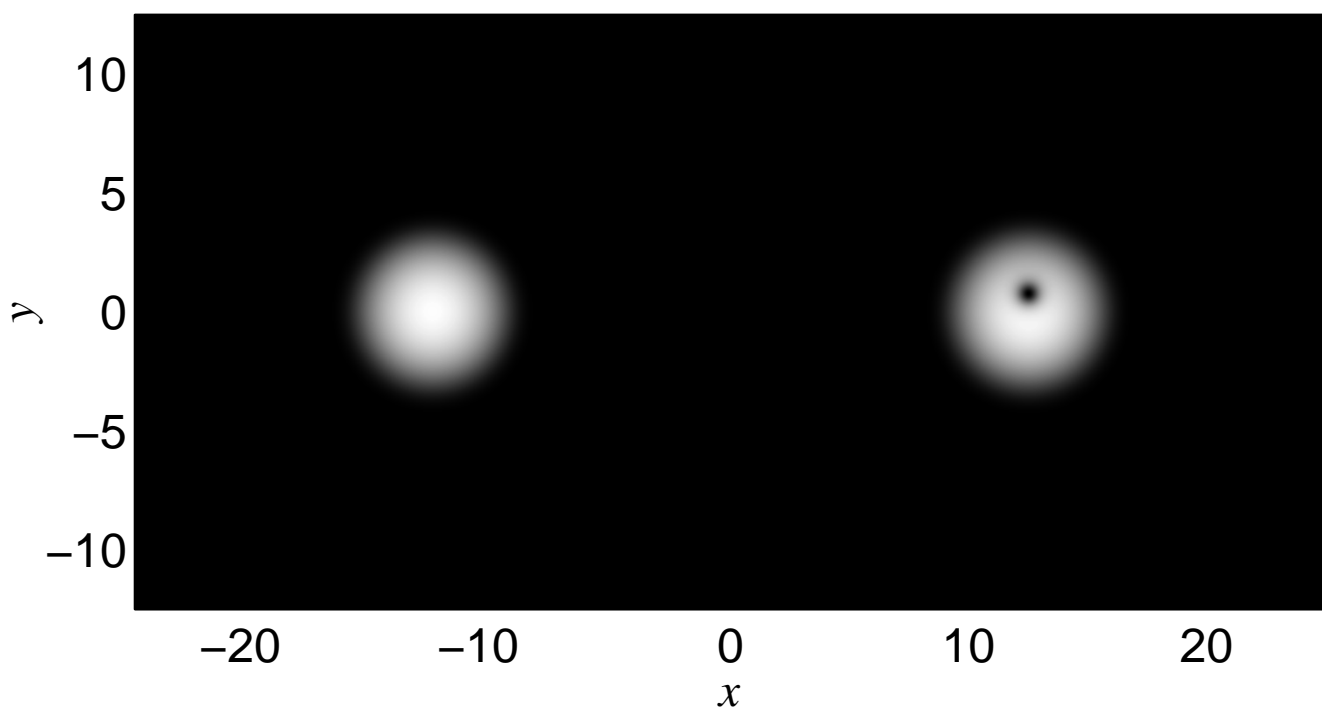


Fig. 4

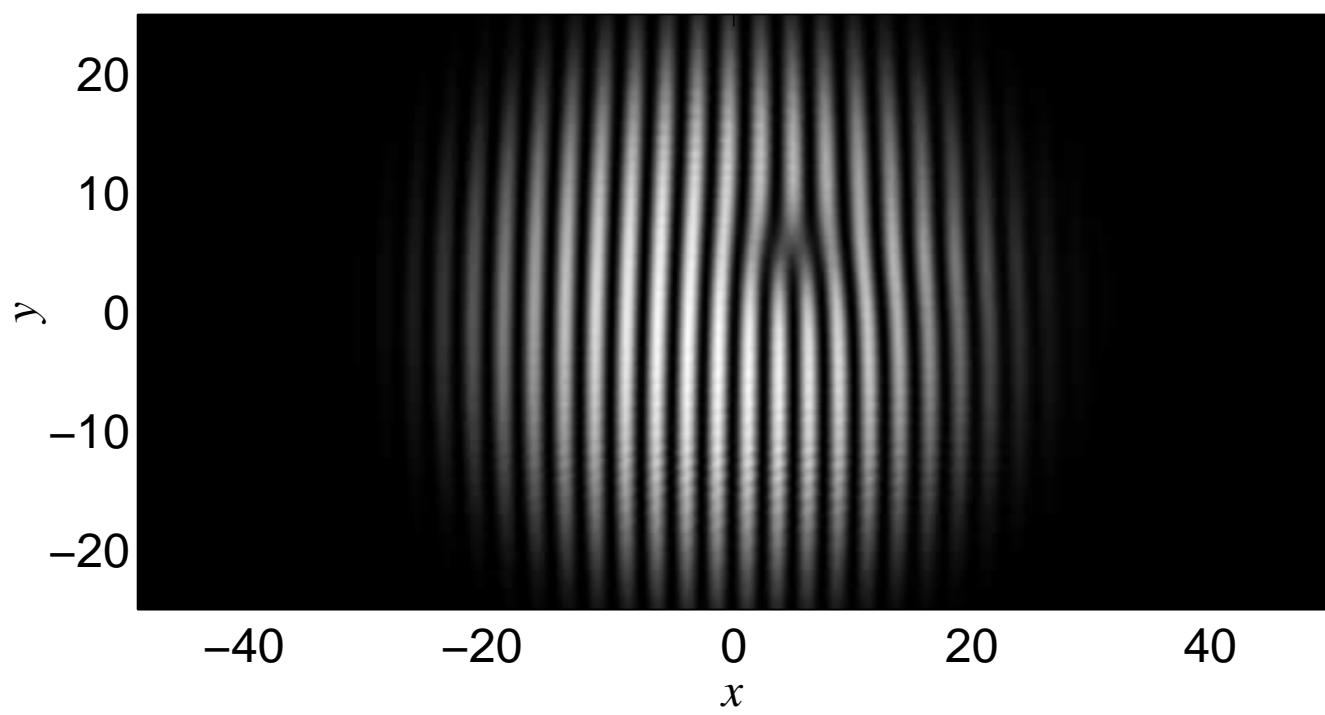


Fig. 5

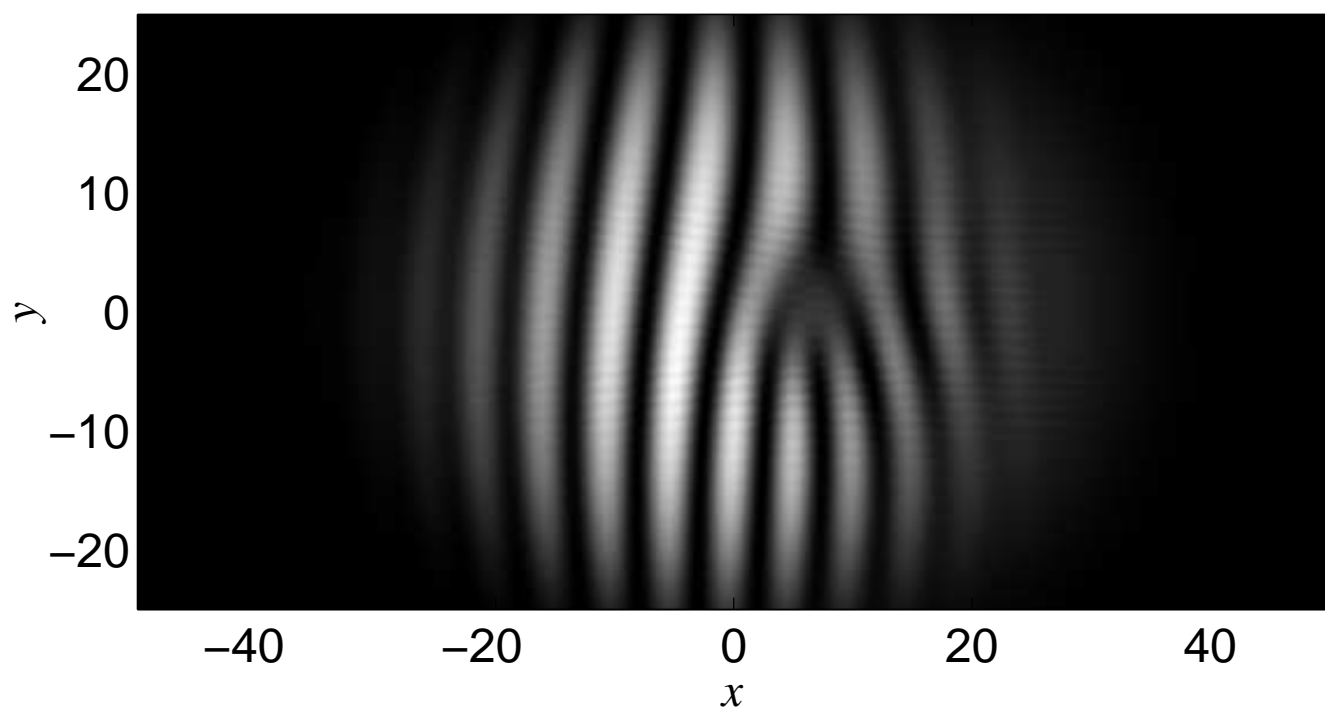


Fig. 6